An Analysis of Recursive Projection-Aggregation Decoding of Reed-Muller Codes

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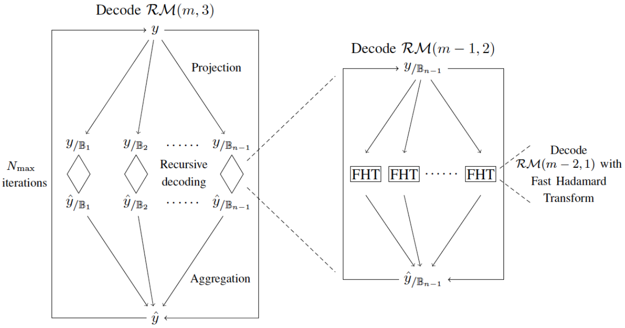
CodeIT: CNI Workshop on Codes, Sequences and Information Theory

Happy 70^{th}, Prof. Vijay Kumar!

What is this talk about?

Consider the family of binary Reed-Muller (RM) codes.



Source: [Ye-Abbe (2020)]

We show that the Recursive Projection-Aggregation (RPA) decoder of [Ye-Abbe (2020)] achieves vanishing error probabilities over the BSC for RM codes of low rate.

Brief background: RM codes

- Fix $m \geq 1$ and consider the points $(x_1, \ldots, x_m)$ of the Boolean hypercube $\{0,1\}^m$.
- Define $x_S := \prod_{i \in S} x_i$, where $S \subseteq [m]$.
- Pick a multilinear polynomial $f = \sum_{S \in \mathcal{S}} x_S$, where $\mathcal{S} \subseteq \mathcal{P}([m])$, with

$$\deg(f) = \max_{S \in \mathcal{S}} |S| \leq r.$$

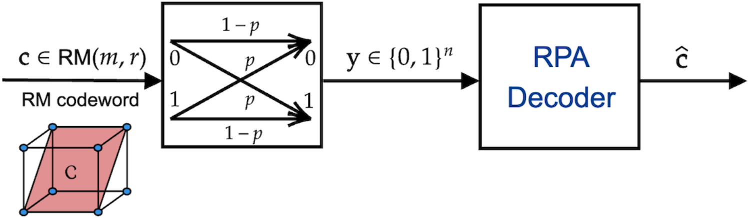
- Evaluate f at all points in $\{0,1\}^m$ in the (lexicographic) order:

$$000\ldots00 \to 000\ldots01 \to 000\ldots10 \to \ldots \to 111\ldots11,$$

  and call the resultant vector $\mathrm{Eval}(f)$. Here, blocklength $n = 2^m$.
- The code $\mathrm{RM}(m,r)$ consists of all $\mathrm{Eval}(f)$, where f is as above.
- $\dim(\mathrm{RM}(m,r)) = \#\{x_S : \deg(x_S) = |S| \leq r\} = \sum_{i=0}^{r} \binom{m}{i} =: \binom{m}{\leq r}$.

Problem setup

Consider the transmission of an RM codeword across a binary symmetric channel (BSC).



Q: How large can the rate be for $\Pr[\widehat{\mathbf{c}} \neq \mathbf{c}] \xrightarrow{N\to\infty} 0$?

TL;DL: The parameter r can grow $\approx$ logarithmically in m

Placing things in context

- [Reed (1954)] provided a decoder capable of correcting $< \frac{d_{\text{min}}}{2}$ adversarial errors.
- For first-order ($r = 1$) RM codes, [Green (1966)] and [Be'ery-Snyders (1986)] described efficient ML decoding, via the Fast Hadamard Transform (FHT).
- For second-order RM codes, see [Sidel'nikov-Pershakov (1992)] and [Sakkour (2005)] for decoders that work well at moderate blocklengths.
- Provably good error guarantees over the BSC for constant r obtained via [Dumer (2004, 2006), Dumer-Shabunov (2006)]
- More recently, data-driven decoding methods have been explored [Jamali et al. (2023)]

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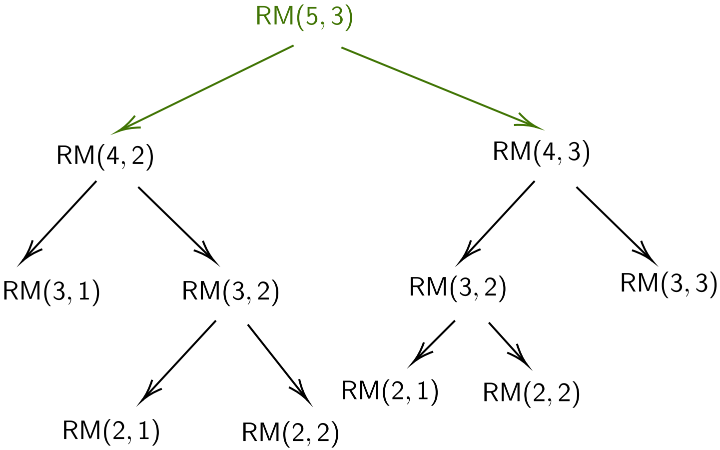
Simulations [Ye-Abbe (2020), Li et al. (2021), Fathollahi et al. (2021)] demonstrate good performance of the RPA decoder for "low" r values.

What is RPA decoding?

Via the Plotkin decomposition, Recursive(ly) Project using all one-dimensional subspaces



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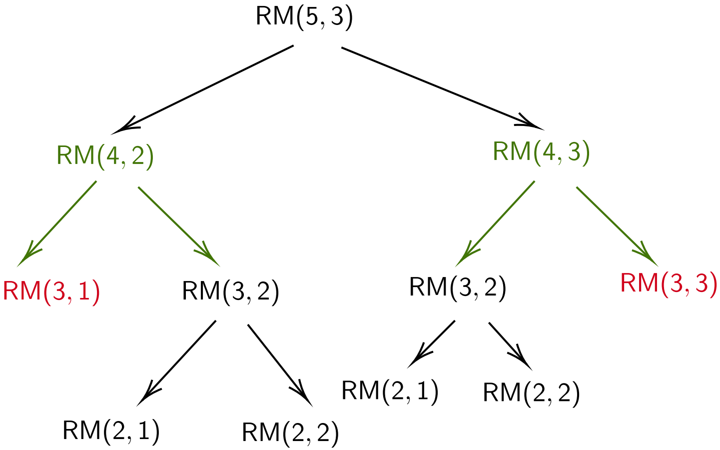

ML decode at red codes

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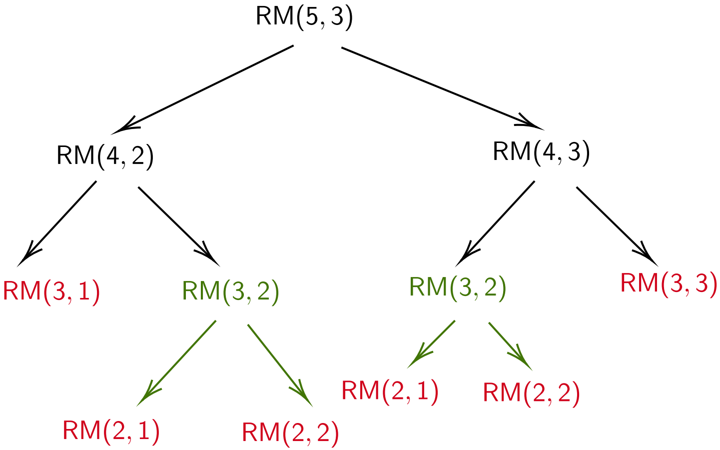

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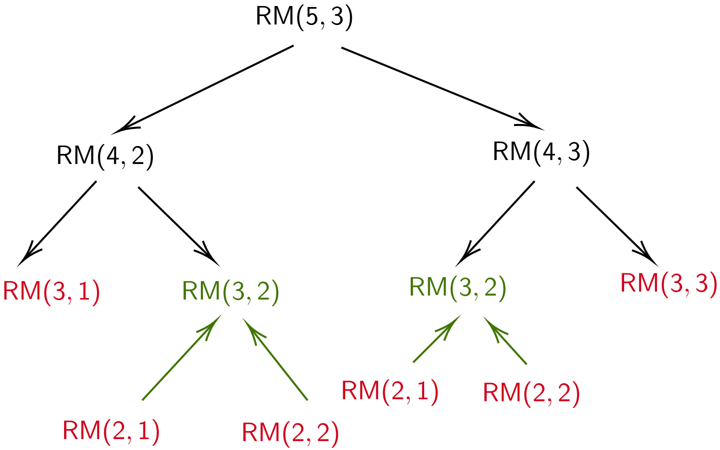
What is RPA decoding?

Via the Plotkin decomposition, Recursive(ly) Project using all one-dimensional subspaces and Aggregate decoded estimates



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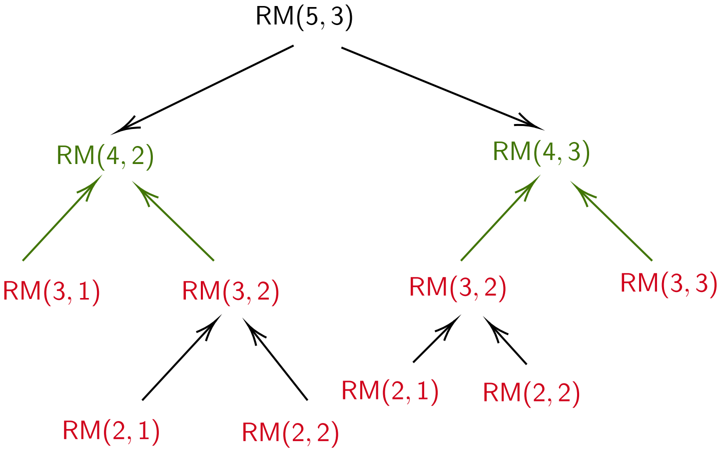

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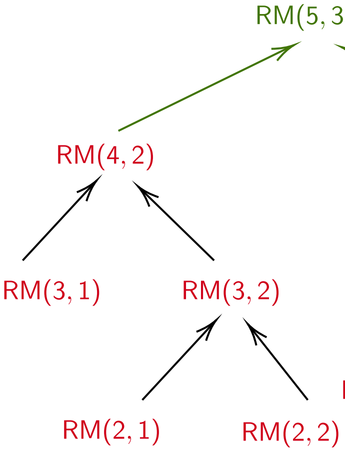


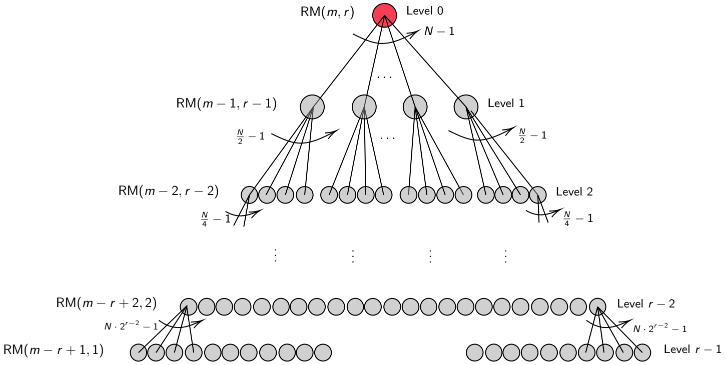
Repeat procedure over several iterations

A closer view of PA

Projection: For a received $\mathbf{Y}$, pick each one-dim. subspace $\mathbb{B}_i$ in turn and construct

$$\mathbf{Y}_{/\mathbb{B}_i} := \left(Y_{/\mathbb{B}_i}(T) : \; T \in \{0,1\}^m/\mathbb{B}_i \right),$$

where $Y_{/\mathbb{B}_i}(T) := \bigoplus_{\mathbf{b}\in\mathbb{B}_i} Y_{\mathbf{x}\oplus\mathbf{b}}$.

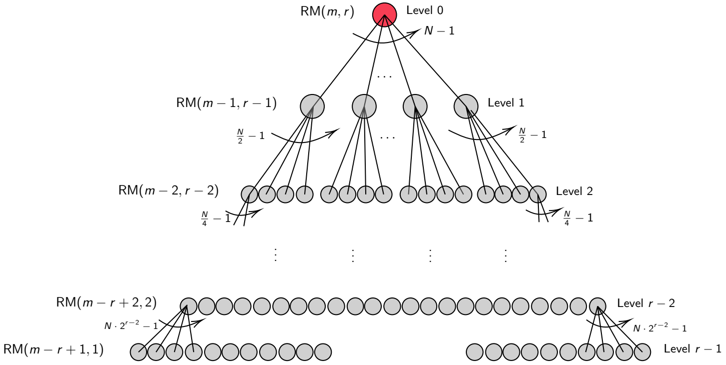


A closer view of PA

Aggregation: At a "parent node," for each $\mathbf{x} \in \{0,1\}^m$, compute

$$\phi(\mathbf{x}) = \sum_{i=1}^{N-1} \mathbb{1}\{Y_{/\mathbb{B}_i}([\mathbf{x}+\mathbb{B}_i]) \neq \widehat{Y}_{/\mathbb{B}_i}([\mathbf{x}+\mathbb{B}_i])\},$$

where $\widehat{Y}_{/\mathbb{B}_i} \leftarrow$ decoded estimate of a "child node". Flip $Y_{\mathbf{x}}$, if $\phi(\mathbf{x}) > \frac{N-1}{2}$.



Our main result(s)

Let $\overline{p} := \frac{1}{2} \cdot (1 - (1-2p)^{2^{r-2}})$ and $\eta(\overline{p}) := \frac{1}{2} \cdot (1 - 4\overline{p}(1-\overline{p}))$.

Theorem

For any $0 < \epsilon < \eta(\overline{p})$, we have that for $r \geq 2$, using one-dimensional subspaces for projection,

$$P_{\mathrm{err}}^{\mathrm{RPA}}(\mathrm{RM}(m,r)) \leq 32N^{r+1} \cdot \exp\left(-2^{-r-1}N\epsilon^2\right).$$

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Corollary

For any $0 < \overline{c} < \frac{\log 2}{\log\left(\frac{1}{1-2p}\right)}$, we have that for all $r \leq \log(\overline{c}m)$,

$$\lim_{m\to\infty} P_{\mathrm{err}}^{\mathrm{RPA}}(\mathrm{RM}(m,r)) = 0.$$

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Further, let $p^{(j)} := \frac{1}{2} \cdot \left(1 - (1-2p)^{2^j}\right)$.

Theorem

For any $0 < \epsilon < \eta(\overline{p})$, we have that for $r \geq 2$, using k-dimensional subspaces for projection, where $k|(r-1)$,

$$P_{\text{err}}^{\text{RPA}}(\text{RM}(m,r)) \leq 64N^3 \cdot n_{k,m}^{\frac{r-1}{k}} \cdot \exp\left(-\ln\left(\frac{1-p^{(r-k-1)}}{p^{(r-k-1)}}\right) \cdot 2^{-r-1-k} N\epsilon^2\right).$$

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However, the rate guarantee $r \lessapprox \log m$ does not change ...

Key steps/ideas in analysis

1. Via symmetry, it suffices to focus on $\mathbf{c} = \mathbf{0}$, since

$$P_{\text{err}}^{\text{RPA}}(\text{RM}(m, r)) = P_{\text{err}, \mathbf{0}}^{\text{RPA}}(\text{RM}(m, r)).$$

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2. Restrict attention to the event where **one** iteration of RPA is sufficient for convergence:
$$\mathbf{Y} \xrightarrow{\text{RPA}} \mathbf{0} \xrightarrow{\text{RPA}} \mathbf{0} \xrightarrow{\text{RPA}} \dots \xrightarrow{\text{RPA}} \mathbf{0}.$$

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FHT Analyze error probability for the "base" case ($r = 1$).

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We hence embark on the analysis of FHT and Agg for order-2 RM codes.

Elaborating on Step FHT

Via the simple map $x \mapsto (-1)^x$, for $x \in \{0,1\}$, we can view binary strings as ± 1-vectors.

- For $\mathbf{s} \in \{0,1\}^m$, let $\chi_{\mathbf{s}} := ((-1)^{\mathbf{x}\cdot\mathbf{s}} : \ \mathbf{x} \in \{0,1\}^m)$. Then,

$$\{\mathbf{c} \in \mathrm{RM}(m,1)\} \mapsto \{\pm\chi_{\mathbf{s}} : \ \mathbf{s} \in \{0,1\}^m\}.$$

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- Then, it can be argued that

$$\mathrm{FHT}(\mathbf{Y}_{/\mathbb{B}_i}) \equiv \underset{\pm\chi_{\mathbf{s}}}{\mathrm{argmax}} \langle \mathbf{Y}_{/\mathbb{B}_\mathbf{i}}, \pm\chi_{\mathbf{s}} \rangle.$$

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- Via simple concentration of the inner products, this yields

Theorem

For all $\epsilon < \eta(p)$*,*

$$\Pr[\mathrm{FHT}(\mathbf{Y}_{/\mathbb{B}_i}) \neq \mathbf{0}] \leq 8N \cdot e^{-\frac{N\epsilon^2}{8}}.$$

See also [Burnashev-Dumer, T-IT 2006]

Elaborating on Step Agg

Conditioned on **all child nodes** being decoded **correctly**,

- aggregation reduces to checking if

$$\phi(\mathbf{x}) = \sum_{i=1}^{N-1} \left(Y_{\mathbf{x}} \oplus Y_{\mathbf{x}+\mathbf{b}_i} \right)$$

  is $<$ or $> \frac{N-1}{2}$.

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- since $\phi(\mathbf{x})$ concentrates around its mean,

$$\text{Flip}(\mathbf{x}) = \mathbb{1}\left\{\overline{\phi}(\mathbf{x}) > \frac{1}{2}\right\} \approx \mathbb{1}\left\{\overline{\phi}_{\infty}(\mathbf{x}) > \frac{1}{2}\right\} = Y_{\mathbf{x}},$$

  for large N, where $\overline{\phi}_{\infty}(\mathbf{x}) := p(1 - Y_{\mathbf{x}}) + (1 - p)Y_{\mathbf{x}}$.

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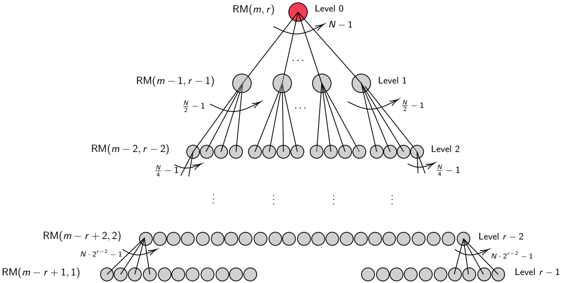
For all $\epsilon < \eta(p)$,

$$\Pr\left[\overline{\mathbf{Y}} = \mathbf{0}\right] = \Pr\left[\text{Flip} = \mathbf{Y}\right] \geq 1 - 32N^3 \cdot e^{-\frac{N\epsilon^2}{8}}.$$

RPA convergence in one iteration!

Putting everything together



Projection-aggregation tree

Via recursive arguments on the projection-aggregation tree, we get

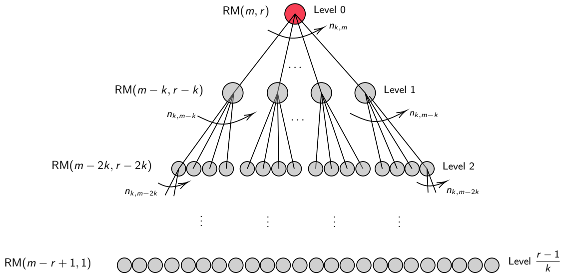
Theorem

For any $0 < \epsilon < \eta(\overline{p})$, we have that for $r \geq 2$, using one-dimensional subspaces for projection,

$$P_{\text{err}}^{\text{RPA}}(\text{RM}(m, r)) \leq 32N^{r+1} \cdot \exp\left(-2^{-r-1}N\epsilon^2\right).$$

Projections using higher dimensional subspaces

- If projections are carried out using k-dimensional subspaces, for $k > 1$, then



  the branching factor is

$$\#k\text{-dim. subspaces of}\{0,1\}^m = \begin{bmatrix} m \\ k \end{bmatrix} := \prod_{i=0}^{k-1} \frac{2^m - 2^i}{2^k - 2^i} =: n_{k,m}.$$

- Then, conditioned on **all children being decoded correctly**, the concentration of $\phi(\mathbf{x})$ must be handled via more sophisticated concentration inequalities.

RPA, but using higher-dim. subspaces

Projection: For a received $\mathbf{Y}$, pick each k-dim. subspace $\mathbb{B}_i$ in turn and construct

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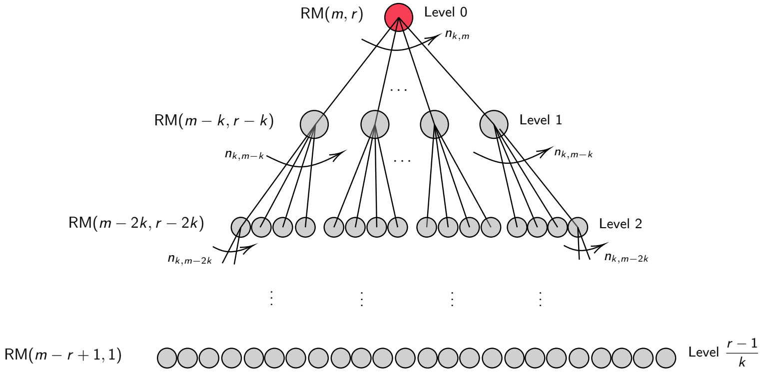

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Aggregation: At a "parent node," for each $\mathbf{x} \in \{0,1\}^m$, compute

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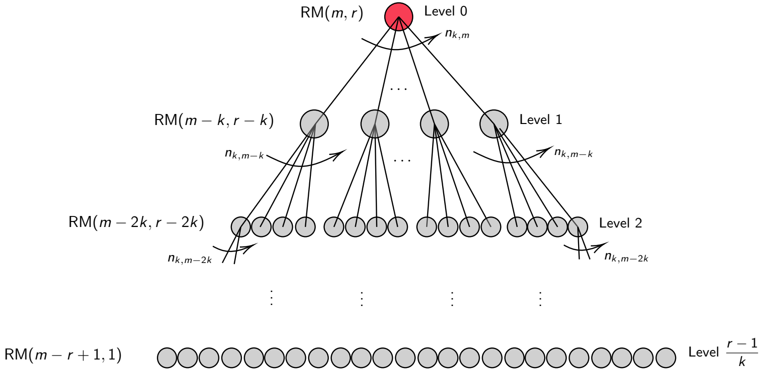

Elaborating on Step Agg

- Conditioned on **all child nodes** being decoded **correctly**, aggregation reduces to checking if

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is $<$ or $>$ $\frac{n_{k,m}}{2}$.

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is $<$ or $> \frac{n_{k,m}}{2}$.

- Concentration of $\phi(\mathbf{x})$ about its mean, however, needs a more sophisticated result:

Theorem (Raginsky-Sason (2018), Thm. 3.4.4)

Let $X_1, \ldots, X_n$ be i.i.d. Ber(q) random variables. Then, for every Lipschitz function $f : \{0,1\}^n \to \mathbb{R}$ with Lipschitz constant c_f, we have for all $\alpha > 0$,

$$\Pr\left[f(X^n) - \mathbb{E}[f(X^n)] > \alpha\right] \le \exp\left(-\ln\left(\frac{1-q}{q}\right) \cdot \frac{\alpha^2}{nc_f^2 \cdot (1-2q)}\right).$$

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is $<$ or $> \frac{n_{k,m}}{2}$.
- An explicit computation of the Lipschitz constant c_f associated with ϕ, seen as a function of $(Y_{\mathbf{x}\oplus\mathbf{b}})$, then results in
$$\text{Flip}(\mathbf{x}) = \mathbb{1}\left\{\overline{\phi}(\mathbf{x}) > \frac{1}{2}\right\} \approx \mathbb{1}\left\{\overline{\phi}_\infty(\mathbf{x}) > \frac{1}{2}\right\} = Y_{\mathbf{x}},$$
for large N, for a suitably defined $\overline{\phi}_\infty(\mathbf{x})$.

Theorem

For all ϵ smaller than a suitable function of p,
$$\Pr\left[\text{Flip} = \mathbf{Y}\right] \geq 1 - \delta_m,$$
for an explicitly computable $\delta_m \xrightarrow{m\to\infty} 0$.

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for an explicitly computable $\delta_m \xrightarrow{m\to\infty} 0$.

Further extensions

1. Can we extend similar analysis to general BMS channels?
2. Can RPA decoding be made more efficient by using a subset of subspaces for projection?

Ongoing work with

 

Dorsa Fathollahi (PhD, Stanford U.), Harshithanjani Athi (PhD, UT Austin), Lalitha Vadlamani (IIIT-H)

Thank You!